

In[*]:=

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SetDirectory[NotebookDirectory[]];
|establece direct... |directorio de cuaderno
Print[
|escribe
"=====
====="];
(** MODELO DE DICKE MODEL**)

(* Classical energy. *)
H[p_, q_, P_, Q_] :=  $\frac{\omega_0}{2} (Q^2 + P^2) - \omega_0 + \frac{\omega}{2} (q^2 + p^2) + 2\gamma q Q \sqrt{1 - \left(\frac{Q^2 + P^2}{4}\right)}$ ;

(* Parameters of the Hamiltonian, energy shell and time of integration. *)
 $\omega = 1$ ;
 $\omega_0 = 1.0$ ;
 $\epsilon = -1.4$ ;
 $\gamma = 2\gamma_c$ ;
T = 5000;
 $\gamma_c = \sqrt{\omega \omega_0} / 2$ ;

(* Dynamical System: Hamilton equations *)
F[{p_, q_, P_, Q_}] :=
{- $\partial_q H[p, q, P, Q]$ ,  $\partial_p H[p, q, P, Q]$ , - $\partial_Q H[p, q, P, Q]$ ,  $\partial_P H[p, q, P, Q]$ };
f =  $\gamma / \gamma_c$ ;
(* Initial condition *)
pi = 0;
(*Pi=0;
Qi=0.707;*)
Pi = 0.6481;
Qi = -1.371;
qs1 = Solve[H[pi, x, Pi, Qi] ==  $\epsilon$ , x];
|resuelve
(*We select one the two roots*)
qi = x /. qs1[[2]];
(*****
INFORMATION *****)
Print["Initial condition in the variables (p,q,P,Q): pi = ",
|escribe
pi, ", qi = ", qi, ", Pi = ", Pi, ", Qi = ", Qi];
|número pi
Print["The system has energy: E = ", H[pi, qi, Pi, Qi], ", and coupling  $\gamma =$ ", f, " $\gamma_c$ "];
|escribe |número e
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Initial condition in the variables (p,q,P,Q): pi = 0, qi = 2.0981, Pi = 0.6481, Qi = -1.371
The system has energy: E = -1.4, and coupling $\gamma = 2.\gamma_c$

In[*]:=

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Print[
|escribe
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"-----
");
Timing[
  Cronometra
  (*Differential equations of the system defined by: p, q, P, Q *)
  Dinam = Table[D[y_k[t], {t, 1}] == F[{y_1[t], y_2[t], y_3[t], y_4[t]}][[k]], {k, 1, 4}];
  (*Initial conditions*)
  CI = {y_1[0] == p_i, y_2[0] == q_i, y_3[0] == P_i, y_4[0] == Q_i};
  (*Jacobian Matrix *)
  JacobianMatrix[f_List, v_List] := Outer[D, f, v];
  (*Variables: y_1=p, y_2=q, y_3=P, y_4=Q*)
  J = JacobianMatrix[F[{y_1[t], y_2[t], y_3[t], y_4[t]}], {y_1[t], y_2[t], y_3[t], y_4[t]}];
  (*Monodromy matrix 4x4*)
  Phi = Table[{y_k[t], y_{k+1}[t], y_{k+2}[t], y_{k+3}[t]}, {k, 5, 20, 4}];
  (*Derivative of the Monodromy*)
  DPhi = Flatten[J.Phi];
  (* Ecuaciones Differential equations of the monodromy matrix *)
  Var = Table[D[y_k[t], {t, 1}] == DPhi[[k - 4]], {k, 5, 20}];
  (*Initial condition of the variational matrix must be the identity *)
  CIVar = Table[y_k[0] == If[Mod[k, 5] == 0, 1, 0], {k, 5, 20}];
  (*sol=NDSolve[Join[Dinam,CI,Var,CIVar],Table[y_k[t],{k,1,20}],{t,0,T},
  StepMonitor-> Sow[x], Method-> Automatic, MaxStepSize->0.1, AccuracyGoal-> 15];*)
  sol = NDSolve[Join[Dinam, CI, Var, CIVar], Table[y_k[t], {k, 1, 20}],
  {t, 0, T}, Method -> {"ExplicitRungeKutta", "DifferenceOrder" -> 8},
  StepMonitor -> Sow[x], MaxSteps -> 10^9, AccuracyGoal -> 30];
  (*error limit*)
  error = 10^-6;
  Do[If[Evaluate[error > Abs[epsilon - H[y_1[t], y_2[t], y_3[t], y_4[t]]] /. sol[[1]], tmax = t],
  {t, 0, T, 1}];
  Print["Numerical solution acceptable at time t: ",
  tmax, " with error: 10^", Log10[error], " of tolerance."];

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T = tmax;
(*Monodromy matrix at time*)
PhiT = Table[{yk[t], yk+1[t], yk+2[t], yk+3[t]}, {k, 5, 20, 4}] /. sol /. t -> T;
      |tabla

(*****
  LYAPUNOV EXPONENT *****
)
VV = Transpose[PhiT[[1]].PhiT[[1]];
      |transposición
λ = (1 / (2 T)) Log[Norm[VV]];
      |lo... |norma

Print["Lyapunov exponent: λ = ", λ];
|escribe

Print[
|escribe
  "-----
  --"];

];
tCPU = %;
Print["Time used: ", tCPU[[1]] / 60, " minutes."];
|escribe

Print[
|escribe
  "=====
  ====="];

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Numerical solution acceptable at time t: 5000 with error: 10⁻⁶ of tolerance.

Lyapunov exponent: λ = 0.051226

Time used: 0.0046875 minutes.

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In[*]:= datOrb = Table[{t, y1[t], y2[t], y3[t], y4[t]} /. sol[[1]] /. t -> i, {i, 0, 300, 0.05}];  
ListPlot[datOrb[[1 ;; -1, {3, 2}]], Joined -> True, AspectRatio -> 1]
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Out[*]=

